

Exponential Growth



and Money



FEDERAL RESERVE NOTE

361663 D



THIS IS LEGAL TENDER
FOR ALL DEBTS, PUBLIC AND PRIVATE

Jacobus Cabral
Secretary of the United States.

SERIES
2006
A



ONE HUNDRED DOLLARS



KF 1836166

Tony M. Santoro
Secretary of the Treasury.

H 353

ONE HUNDRED DOLLARS

FRANKLIN



Interest



Simple Interest

- Mostly used in casual situations
- Also used in the bond market and for some short-term loans



Simple Interest

- Interest is paid **ONLY** on the base amount of money (the principal)
- The same amount of interest is paid each time.



\$100 investment → 3% simple interest

Year	Interest Earned	Total
1	\$3	\$103
2	\$3	\$106
3	\$3	\$109
4	\$3	\$112
5	\$3	\$115



Simple Interest Formulas

$$I = Prt$$

$$A = P + I \text{ or } A = P + Prt$$



You lend your brother \$350. If he pays the loan back 2 years later, how much will he owe you if you charge 5% interest?



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```
350*.05*2
350+35
385
```



An appliance store charges 4% simple interest on purchases that aren't paid immediately. If you buy an \$800 stove, how much interest will you owe if you pay for the purchase 6 months later?



$$r = .04$$

$$P = 800$$

$$t = \frac{1}{2}$$

$$800 * .04 * .5$$

16





A bond matures 20 years after it is purchased. If its value at maturity is \$500 and the bond earned 2% annually, how much was the purchase price?



$$A = P + Prt$$

$$500 = P + P * .02 * 20$$

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$$A = P + Prt$$

$$500 = P + P * .02 * 20$$

$$500 = 1P + .4P$$

$$500 = 1.4P$$

$$\$357.14 = P$$



Simple interest
is NOT
exponential growth.



Compound Interest

- Used in most financial transactions
- Interest is paid both on the principal AND on interest that has already been earned



\$100 investment → 3% compound interest

Year	Interest Earned	Total
1	\$3	\$103.00
2	\$3.09	\$106.09
3	\$3.18	\$109.27
4	\$3.28	\$112.55
5	\$3.38	\$115.93



Compound Interest

- Each year you earn more interest than the year before
- Growth is exponential.



Compound Interest Formula

$$A = P \left(1 + \frac{r}{n} \right)^{(nt)}$$



$$A = P \left(1 + \frac{r}{n} \right)^{(nt)}$$

A = Accumulation (final amount)

P = Principal (starting amount)

r = Rate of interest (decimal)

t = Time (years)



$$A = P \left(1 + \frac{r}{n} \right)^{(nt)}$$

n = number of times compounded
per year



Annually = 1

Semiannually = 2

Quarterly = 4

Monthly = 12

Weekly = 52

Daily = ??????



Daily

→ In modern times usually
365

→ Traditionally 360

→ Sometimes 366 or 365.25



Families will supposedly get a \$500 payment from the government for each child they have as an economic stimulus due to the Coronavirus pandemic.



Suppose your parents let you invest your \$500. You find an account that will pay 6% interest, compounded quarterly. How much money will you have 50 years from now?



$$A = 500 \left(1 + \frac{.06}{4} \right)^{(4*50)}$$

$$500(1 + .06 \div 4)^{(4 \times 50)} = 9821.51431974$$

Calculator interface showing the input expression: $500(1 + .06 \div 4)^{(4 \times 50)}$. The display shows "Ans = 0".

Rad	Deg	x!	()	%	CE
Inv	sin	ln	7	8	9	÷
π	cos	log	4	5	6	×
e	tan	√	1	2	3	-
Ans	EXP	x ^y	0	.	=	+

Calculator interface showing the result: 9821.51431974 .

()	%	AC
7	8	9	÷
4	5	6	×



$$A = 500 \left(1 + \frac{.06}{4} \right)^{(4*50)}$$

```
500(1+.06/4)^(4*  
50)  
9821.51432
```

```
1+.06/4  
1.015  
Ans^200  
19.64302864  
Ans*500  
9821.51432
```



$$A = 500 \left(1 + \frac{.06}{4} \right)^{(4*50)}$$

\$9821.51





A \$10,000 investment earns 9% interest compounded daily. How much will it be worth in 15 years?



$$A = 10000 \left(1 + \frac{.09}{365} \right)^{(365*15)}$$

```
10000(1+.09/365)  
^(365*15)  
38567.83667
```



$$A = P \left(1 + \frac{r}{n} \right)^{(nt)}$$

