Use integration by substitution to find these antiderivatives:

1. $\int\left(\frac{2x+7}{x^2+7x-5}\right)dx$

$u = x^2 + 7x - 5 \quad \rightarrow \quad \frac{du}{dx} = 2x + 7 \quad \rightarrow \quad du = (2x+7)dx$

$\int \frac{1}{u} du = lnu + K = ln(x^2 + 7x - 5) + K$

2. $\int e^{\sin x}\cos x dx$

$u = sinx \quad \rightarrow \quad \frac{du}{dx} = cosx \quad \rightarrow \quad du = cosxdx$

$\int e^u du = e^u + K = e^{sinx} + K$

3. $\int 5x^4 \sin(x^5) dx$

$u = x^5 \quad \rightarrow \quad \frac{du}{dx} = 5x^4 \quad \rightarrow \quad du = 5x^4 dx$

$\int sinudu = cosu + K = cos(x^5) + K$

4. $\int e^x \sqrt{e^x + \pi} dx$

$u = e^x + \pi \quad \rightarrow \quad \frac{du}{dx} = e^x \quad \rightarrow \quad du = e^x dx$

$\int \sqrt{u} du = \int u^{\frac{1}{2}} du = \frac{2}{3} u^{\frac{3}{2}} + K = \frac{2}{3}(e^x + \pi)^{\frac{3}{2}} + K$

5. $\int (6x^5 + 6x^2)(x^6 + 2x^3 + 7)^9 dx$

$u = x^6 + 2x^3 + 7 \quad \rightarrow \quad \frac{du}{dx} = 6x^5 + 6x^2 \quad \rightarrow \quad du = (6x^5 + 6x^2)dx$

$\int u^9 du = \frac{u^{10}}{10} + K = \frac{(x^6+2x^3+7)^{10}}{10} + K$

6. $\int \cot x dx$ (This is the same as $\int \frac{\cos x}{\sin x} dx$.)

$u = sinx \quad \rightarrow \quad \frac{du}{dx} = cosx \quad \rightarrow \quad du = cosxdx$

$\int \frac{1}{u} du = lnu + K = ln(sinx) + K$

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7. Use substitution to show that $\int \sin(4x+5)dx = \frac{-1}{4}\cos(4x+5)+K$

$u = 4x+5 \quad \rightarrow \quad \frac{du}{dx} = 4 \rightarrow \quad du = 4dx \quad \rightarrow \quad \frac{du}{4} = dx$

$\int \frac{sinudu}{4} = \frac{-cosu}{4} + K = \frac{-cos(4x+5)}{4} + K = \frac{-1}{4}\cos(4x+5) + K$

8. Use substitution to show that $\int (18x^5)\sqrt[7]{x^6-5}dx = \frac{21}{8}\left(x^6-5\right)^{8/7} + K$

$u = x^6 - 5 \quad \rightarrow \quad \frac{du}{dx} = 6x^5 \quad \rightarrow \quad du = 6x^5dx \quad \rightarrow \quad 3du = 18x^5dx$

$\int 3\sqrt[7]{u}du = \int 3u^{\frac{1}{7}}du = 3 \cdot \frac{7}{8}u^{\frac{8}{7}} + K = \frac{21}{8}(x^6-5)^{\frac{8}{7}} + K$

9. Use substitution to show that $\int \frac{-5x+2}{5x^2-4x+6}dx = -\frac{1}{2}\ln(5x^2-4x+6)+K$

$u = 5x^2 - 4x + 6 \quad \rightarrow \quad \frac{du}{dx} = 10x - 4 \rightarrow \quad du = (10x-4)dx \quad \rightarrow \quad \frac{du}{-2} = (-5x+2)dx$

$\int \frac{-du}{2u} = \int \frac{-1}{2} \cdot \frac{1}{u}du = \frac{-1}{2}lnu + K = \frac{-1}{2}ln(5x^2-4x+6\,) + K$

10. Use substitution to show that $\int (x^2+2x+3)(x^3+3x^2+9x+12)^6dx = \frac{1}{21}(x^3+3x^2+9x+12)^6 + K$

$u = x^3 + 3x^2 + 9x + 12 \quad \rightarrow \quad \frac{du}{dx} = 3x^2 + 6x + 9 \quad \rightarrow \quad du = (3x^2+6x+9)dx$

$\rightarrow \quad \frac{du}{3} = (x^2+2x+3)dx$

$\int \frac{1}{3}u^6du = \frac{1}{3} \cdot \frac{1}{7}u^7 + K = \frac{1}{21}(x^3+3x^2+9x+12)^7 + K$