NAME:____________________________
Period:_____ Date:________________

Calculus

PHYSICAL APPLICATIONS OF DERIVATIVES

Circle the correct answer.

1. What information does the **ORIGINAL FUNCTION** tell you?

   Acceleration Position Velocity

2. What information does the **FIRST DERIVATIVE** tell you?

   Acceleration Position Velocity

3. What information does the **SECOND DERIVATIVE** tell you?

   Acceleration Position Velocity

4. Which function would answer the question "When does the object reach its highest point?"

   Original Function 1st Derivative 2nd Derivative
   Set 1st derivative equal to 0

5. Which function would answer the question "How fast is the object moving after 9 seconds?"

   Original Function 1st Derivative 2nd Derivative

6. Which function would answer the question "When does the object return to the ground?"

   Original Function 1st Derivative 2nd Derivative
   Set original = 0

7. Which function would answer the question "When is the object moving at –75 feet/second?"

   Original Function 1st Derivative 2nd Derivative
   Set 1st derivative = -75

8. Which function would answer the question "When does the object 30 feet above the surface of the earth?"

   Original Function 1st Derivative 2nd Derivative
   Set original = 30
   (… and the question should say "When IS the object …")

9. Which function would answer the question "How high is the object after 2.75 seconds?"

   Original Function 1st Derivative 2nd Derivative

10. **THINK CAREFULLY:** Which function would answer the question "When is the velocity greatest (or least)?"

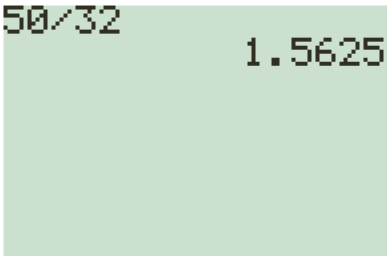
    Original Function 1st Derivative 2nd Derivative
    You're trying to find the maximum velocity, so you do the derivative of velocity.

Physical Applications of Derivatives – Page 2

Now answer these questions.
(In the process you will learn about Mr. Burrow's high school classmates.)

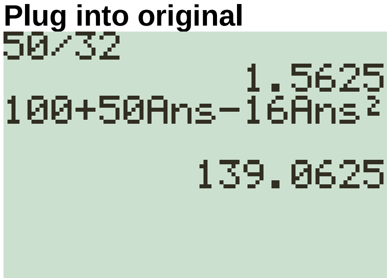
11. Rusty is standing on a cliff 100 feet above the surface of the ocean. He throws a seashell into the air with an initial velocity of 50 feet/second. Because he is very intelligent, Rusty knows the position of the seashell is given by the function $s(t) = 100 + 50t - 16t^2$.
(Rusty was one of Mr. Burrow's best friend in high school.)

a. When is the seashell at its highest point?
Set derivative = 0
$\mathbf{50 - 32t = 0}$



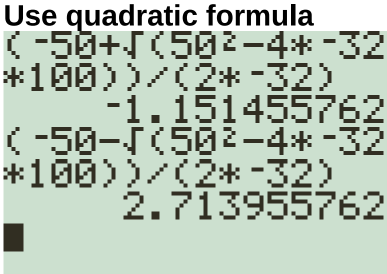
1.5625 seconds

b. How high is the seashell at its highest point?
Plug into original



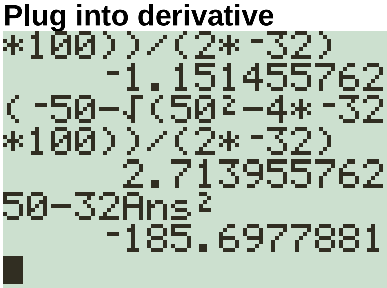
139.0625 feet

c. When does the seashell land on the beach below?
Set original = 0
$\mathbf{100 + 50t - 32t^2 = 0}$
Use quadratic formula



The positive answer is <u>2.7 seconds</u>

d. How fast is the seashell travelling when it lands on the beach?
Plug into derivative

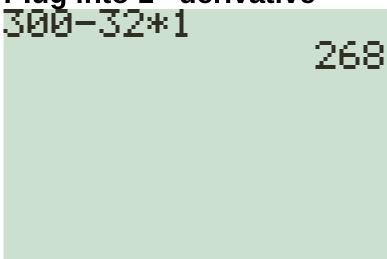


It's going downward at 185.7 $\mathbf{{}^{ft}/_{sec}}$

12. Bounyenne is golfing. When her club hits the ball, it propels her ball through the air with so much upward force that the vertical position of her ball is given by the function $s(t) = 300t - 16t^2$. Unfortunately, there is so much wind that her ball doesn't move forward at all. It just goes up and then comes right back down.
(Bounyenne wouldn't have actually been golfing. She was a Laotian immigrant who was wheelchair-bound.)

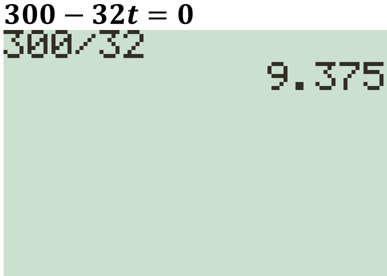
a. The instantaneous acceleration is constant for this problem. What is it?
Second derivative → -32 $^{ft}/_{sec^2}$

b. What is the velocity after 1 second?
Plug into 1st derivative



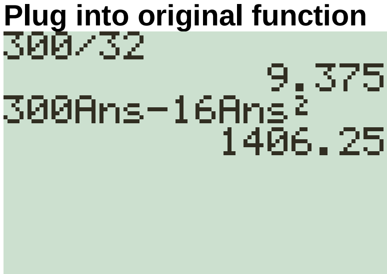
Going up at 268 $^{ft}/_{sec}$

c. When does the ball stop rising and start falling?
Same as "When is it at its highest point?"
Set derivative = 0
$\mathbf{300 - 32t = 0}$



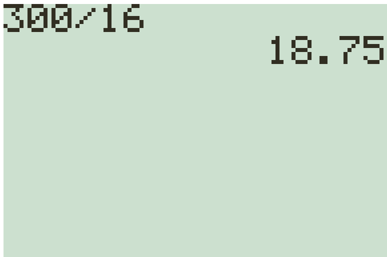
9.375 seconds

d. What is the highest point the ball reaches? (Yes, it's a stupid answer.)
Plug into original function



1406.25 feet, which is about ¼ mile

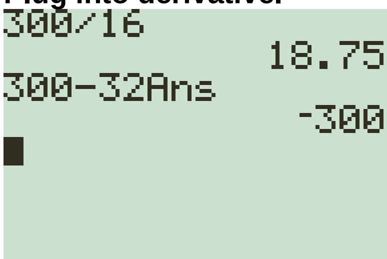
e. When will the ball return to the ground?
Set original = 0
$\mathbf{300t - 16t^2 = 0}$
$\mathbf{t(300 - 16t) = 0}$
Possible answers are 0 and $\frac{300}{16}$



18.75 seconds

NOTE: Whenever something starts on the ground, it will always rise and fall for the same amount of time. 18.75 is exactly double 9.375, the answer to Part c.

f. What is the velocity when the ball hits the ground?
Plug into derivative.

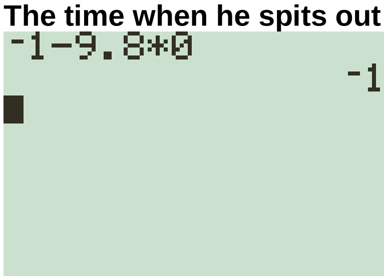


Going down at 300 feet per second

NOTE: In problems that start and end on the ground, the initial upward velocity (the 300 in the original function) and the final velocity are always the same amount, but in opposite directions.

13. Ed's mouth is 1.5 meters off the ground. When he spits his gum out, it travels with an initial downward velocity of 1 meter/second. Thinking quick, Ed calculates that the position of his gum at any given time is given by the function $s(t) = 1.5 - t - 4.9t^2$.
(Ed was another of Mr. Burrow's best friends in high school, and someone he still occationally keeps in touch with.)

a. What is the instantaneous velocity at the second Ed spits the gum out of his mouth?
The time when he spits out the gum is 0 seconds. Plug into derivative

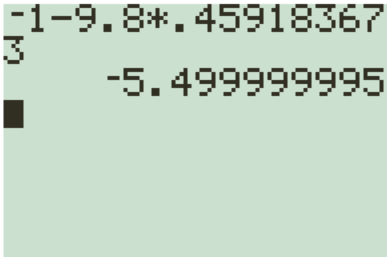


-1 m/s

NOTE: This is always the linear coefficient, the number by "t" in the original function.

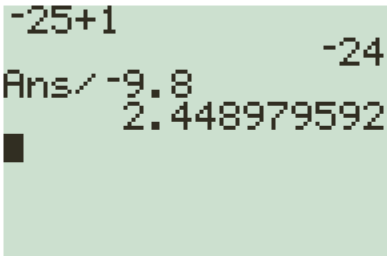
b. What is the instantaneous acceleration at the second Ed spits the gum out of his mouth?
Second derivative → -9.8 m/s^2 (This is the standard acceleration of gravity in meters.)

c. Using the quadratic formula, you can figure out that the gum hits the pavement at approximately .459183673 seconds. Find the velocity at this time.
Plug into derivative.



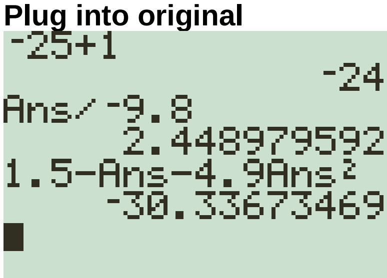
going downward at 5½ meters per second

d. Suppose instead of landing on the ground, the gum falls into a gutter and keeps on falling. After how long will the velocity be –25 meters/second?
Set derivative = -25
$\mathbf{-1 - 9.8t = -25}$



about 2.4 seconds

e. How far below ground level will the gum be when the velocity is –25 meters/second?
Plug into original



about 30.3 feet below ground level

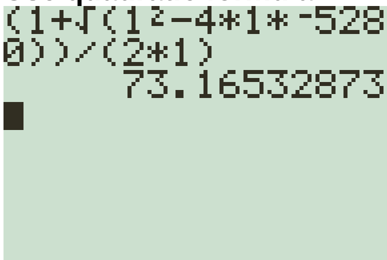
Physical Applications of Derivatives – Page 3

14. Kaila is roller-skating down a hill in Charleston, South Carolina. The function describing her position is $s(t) = t - t^2$.
(Kaila, pronounced "KYE-luh", was voted "most bizarre" in Mr. Burrow's senior yearbook. She was originally from Charleston and could never stop talking about it. Mr. Burrow, by the way, was voted "most likely to succeed"—not sure if that happened or not.)

a. What formula will tell Kaila's velocity?
First derivative
$\mathbf{-1 - 2t}$

b. After travelling one mile (5280 feet), Kaila reaches the bottom of the hill and plunges into Charleston Harbor. (Her position is then –5280 feet.) How long has she been skating?
Set original = -5280
$\mathbf{-5280 = t - t^2}$ **… or** $\mathbf{t^2 - t - 5280 = 0}$

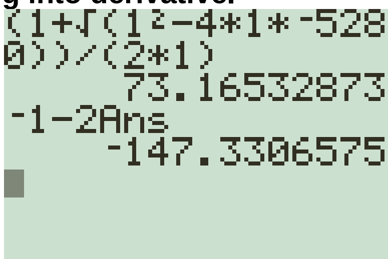
Use quadratic formula



73.2 seconds

NOTE: Since this is the positive answer, we don't need to find the negative one.

c. How fast is she moving when she plunges into the harbor?
Plug into derivative.

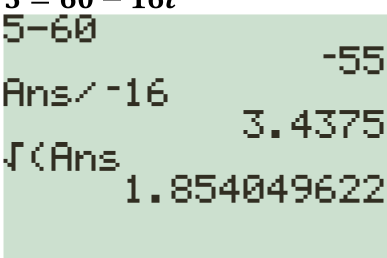


-147.3 ft per sec

d. What is her rate of acceleration?
Second derivative → -2 ft/sec per sec

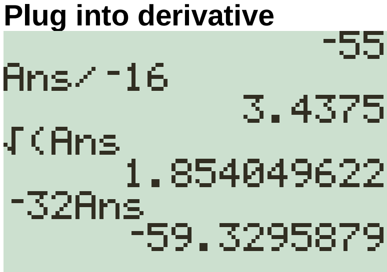
15. Bryan was standing under a 60-foot oak tree. Suddenly an acorn fell downward and hit him on the head. Its position function was $s(t) = 60 – 16t^2$.
(Bryan lived just down the street from Mr. Burrow. He became a real estate agent and now is the majority owner of a minor league baseball team franchise.)

a. If Bryan was 5 feet tall, how long did it take the acorn to land on his head?
Set original function = 5
$\mathbf{5 = 60 - 16t^2}$



about 1.85 seconds

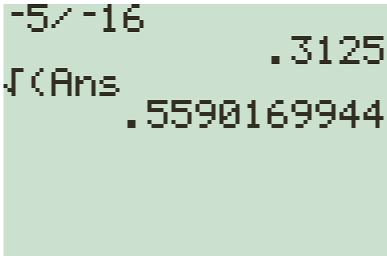
b. How fast was the acorn travelling when it hit Bryan on the head?
Plug into derivative



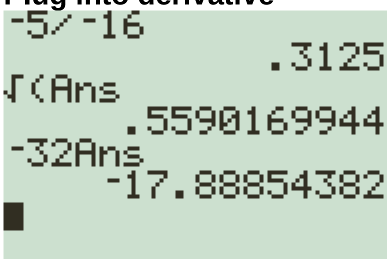
downward at 59.3 feet per second

c. What was the acorn's acceleration?
Second derivative → -32 ft/sec^2

The acorn causes Bryan to pass out. He falls down, and the position function for his body is $s(t) = 5 – 16t^2$.

d. How long does it take Bryan to hit the ground?
Set original = 0
$\mathbf{0 = 5 - 16t^2}$


Just over ½ second

e. How fast is Bryan travelling when he hits the ground?
Plug into derivative


f. What is Bryan's acceleration?
Second derivative → again -32 ft/sec^2

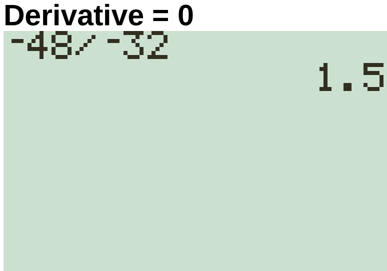
For the following problems, use these rules:

As a general rule, the position function of an object dropped from a height of "h" feet with an initial velocity of "v" feet/second is given by the function $s(t) = h - vt – 16t^2$.

The position of an object thrown upward with an initial velocity of "v" feet/second is given by the function $s(t) = vt – 16t^2$.

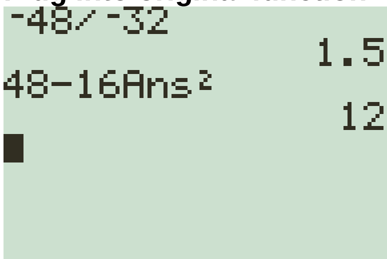
16. Allison throws her baton in the air with an initial velocity of 48 feet/second. Unfortunately, she doesn't catch the baton, and it falls on the ground.
(Allison was literally the "girl next door" when Mr. Burrow was growing up. She was a baton twirler in high school.)

a. Use one of the rules above to write a position function for this problem.
Since no initial height is given in the problem, we'll assume the baton starts on the ground.
$\mathbf{s(t) = 48 - 16t^2}$

b. When is the baton at its highest point?
Derivative = 0


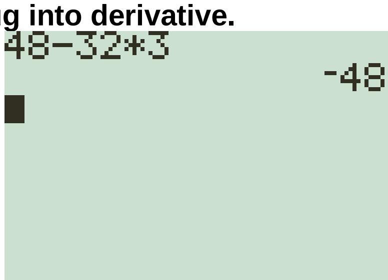
1.5 seconds

c. How high is the baton's highest point?
Plug into original function.



d. When does the baton hit the ground?
Set original = 0 OR just double when it reaches the highest point
$\mathbf{0 = 48t - 16t^2}$
$\mathbf{0 = t(48 - 16t)}$
t = 0 or 3 … So, 3 seconds

e. What is the velocity when the baton hits the ground?
Plug into derivative.



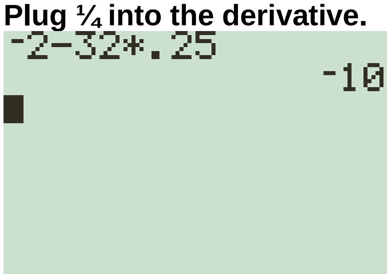
-48 ft/sec (again the opposite of the initial)

Physical Applications of Derivatives – Page 4

17. Jeff slam-dunks a basketball from 11 feet above the court. The initial downward force is 2 feet/second. **(Jeff probably was the best basketball player for the Mt. Pleasant Panthers. However Mr. Burrow's class was not very athletic overall. No one on the team could have dunked the ball.)**

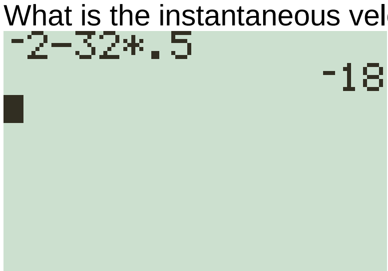
a. Write a position function.
$\mathbf{s(t) = 11 - 2t - 16t^2}$

b. What is the instantaneous velocity after ¼ second?
Plug ¼ into the derivative.



going downward at 10 ft/sec

c. What is the instantaneous velocity after ½ second?

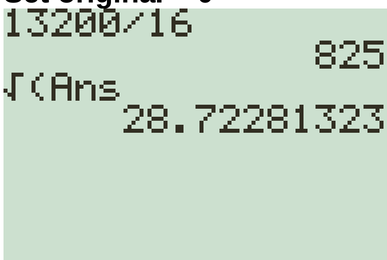


going downward at 18 ft/sec

18. Patty is flying in an airplane 13,200 feet in the air. Suddenly a bolt drops off her plane and drops to the earth. The initial downward velocity is zero.
(Patty served in the Air Force and then became one of the first women to be a long-distance pilot for United Airlines.)

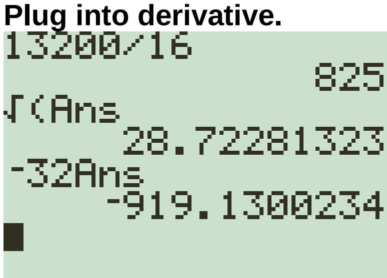
a. Write a position function.
$\boldsymbol{s(t) = 13200 - 16t^2}$

b. How long does it take the bolt to hit the ground?
Set original = 0



just under 29 seconds

c. What is the velocity when the bolt hits the ground?
Plug into derivative.



-919.1 ft/sec

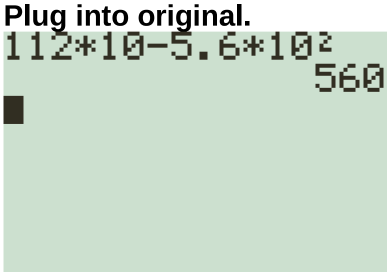
NOTE: In the real world forces like air resistance will affect this velocity, which would actually be significantly slower than this answer.

Now do these problems.

19. When the Viking spacecraft landed on Mars, it threw a rock into the Martian atmosphere. The height of the rock (in feet) at "t" seconds after being thrown is described by the function $s(t) = 112t - 5.6t^2$ (… a true formula!)
NOTE: the "t^2" term has a different coefficient, because the force of gravity on Mars is less than it is on earth.

a. After how many seconds does the rock reach its maximum height?
Derivative = 0
$\boldsymbol{112 - 11.2t = 0}$ **→ 10 seconds**

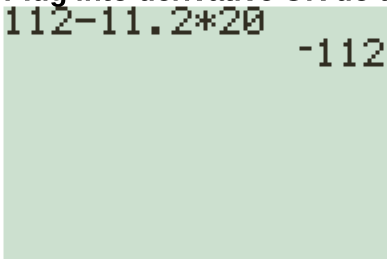
b. How many feet above the surface of Mars is the rock when it is at its highest point?
Plug into original.



560 feet

c. When will the rock hit the surface of Mars?
Set original = 0 OR just double when it's at its maximum height → 20 seconds

d. What is the instantaneous velocity (in feet/second) at the time it hits the surface of Mars?
Plug into derivative OR do the opposite of the initial velocity.



downward at 112 ft/sec

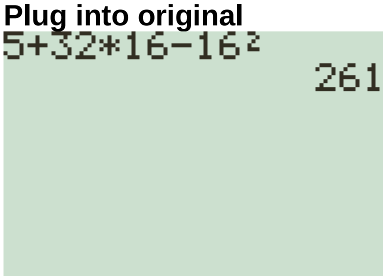
e. What is the instantaneous acceleration (in feet/second^2) of the rock at the time it hits the surface of Mars?
Second derivative → -11.2 ft/sec^2

20. Annabel rolls a ball up a ramp. The position of the ball at any second is given by the function $s(t) = 5 + 32t - t^2$.
(Annabel was actually a friend of Mr. Burrow's in college, rather than high school. She was a physics major, but he hasn't heard from her since he left UNI.)

a. What formula would give you the velocity of the ball?
First derivative
$\mathbf{32 - 2t}$

b. After how many seconds does the ball start rolling backwards (stops rolling up the ramp, and starts rolling back down)?
Same as "highest point", so set derivative = 0 → 16 seconds

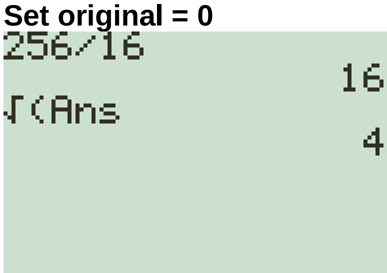
c. What is the furthest distance up the ramp the ball will go?
Plug into original



261 (units not given in the problem)

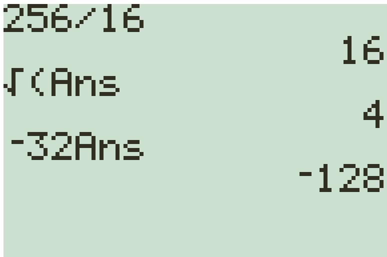
21. A bomb is dropped from a height of 256 feet. The function for its position is therefore $s(t) = 256 - 16t^2$.

a. When will the bomb hit the ground?
Set original = 0



4 seconds

b. How fast will the bomb be falling when it hits the ground?
Plug into derivative.



-128 ft/sec